Key

Unit 9 Day 1 Notes on Simplifying Radicals $\sqrt{}$

Radical – the generic name for square roots, cube roots, 4th roots, etc.

The process of simplifying radicals is a hold-over from before handheld calculators. It does help to organize us a bit, but also to know where on a number line a number falls (number sense). This topic still shows up on standardized tests and so we will need to show you.

Let's get started:

Discuss: Between what two integers does $\sqrt{56}$ fall? $7^2 = 49$ (too low) and $8^2 = 64$ (too high)

* between 7 and 8

One more time – a refresher on the difference between $x^2 = 9$ and $x = \sqrt{9}$

what squared is 9? ± 3

$x = 3$ only (principal root)

Now, why is the $\sqrt{9} = 3$?

Because $\sqrt{9} = \sqrt{3 \cdot 3} = 3$, right? And what about $\sqrt{16}$? $\sqrt{16} = \sqrt{4 \cdot 4} = 4$.

$\sqrt{3^2}$ $\quad$ $\sqrt{4^2}$

$\sqrt{10 \cdot 10} = 10$

$\sqrt{10^2} = 10$

So, for every PAIR under the square root, you get ONE outside.

Try some:

$\sqrt{5 \cdot 5 \cdot 5 \cdot 5} = \sqrt{5^2 \cdot 5^2} = 5 \cdot 5 = 25$

New rule for $\sqrt{}$ **:** $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

So, really, we have two options

Prime factor vs look for perfect squares

$\sqrt{27} = \sqrt{3 \cdot 3 \cdot 3} = \sqrt{3^2 \cdot 3}$ stay $= 3\sqrt{3}$ vs $\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$

Notice the pattern!

$\sqrt[2]{3^4} = \sqrt{3 \cdot 3 \cdot 3 \cdot 3} = 3 \cdot 3 = 3^2$ $\sqrt{3^2 \cdot 3^2}$	$\sqrt[2]{3^{18}} = 3^9$
$\sqrt[2]{3^8} = \sqrt{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$	$\sqrt{3^{100}} = 3^{50}$

$\sqrt[2]{3^{10}} = 3^5$
$\sqrt[2]{x^n} = x^{n/2}$
$\sqrt{x^6} = x^3$ (actually, No. but it is the best we will do) actually in Alg 2 you will learn it is $\lvert x^3 \rvert$
$\sqrt{x^4} = x^2$
$\sqrt{x^9} = \sqrt{x^8 \cdot x} = \sqrt{x^8} \cdot \sqrt{x} = x^4\sqrt{x}$ not even

12 → 6, 2; 6 → 3, 2

Factor tree (not always helpful)

Put it all together 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

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$\sqrt{144} = 12$	$\sqrt{12} = \sqrt{4 \cdot 3}$ $\sqrt{4}\sqrt{3}$ $2\sqrt{3}$ $\sqrt{2 \cdot 2 \cdot 3}$ $2\sqrt{3}$
$-3\sqrt{18} = -3\sqrt{9 \cdot 2}$ $= -3\sqrt{9}\sqrt{2}$ $= -3 \cdot 3 \cdot \sqrt{2}$ $= -9\sqrt{2}$	$5\sqrt{96} = 5\sqrt{16 \cdot 6}$ $= 5\sqrt{16} \cdot \sqrt{6}$ $= 5 \cdot 4 \cdot \sqrt{6}$ $= 20\sqrt{6}$
$a^2\sqrt{a^6}$ $a^2 \cdot a^3$ a^5	$4a^2\sqrt{32a^4b^5} = 4a^2\sqrt{16 \cdot 2 \cdot a^4 \cdot b^4 \cdot b}$ $= 4a^2\sqrt{16} \cdot \sqrt{2} \cdot \sqrt{a^4} \cdot \sqrt{b^4}\sqrt{b}$ $= 4a^2 \cdot 4 \cdot \sqrt{2} \cdot a^2 \cdot b^2 \cdot \sqrt{b}$ $= 16a^4b^2\sqrt{2b}$
$a^5b^7\sqrt{a^2b^{17}c^9} = a^5b^7\sqrt{a^2 \cdot b^{16} \cdot b \cdot c^8 \cdot c}$ $= a^5b^7\sqrt{a^2} \cdot \sqrt{b^{16}} \cdot \sqrt{b} \cdot \sqrt{c^8} \cdot \sqrt{c}$ $= a^5b^7 \cdot a \cdot b^8 \cdot \sqrt{b} \cdot c^4 \cdot \sqrt{c}$ $= a^6b^{15}c^4\sqrt{bc}$	$-4mn\sqrt{88m^3n^7} = -4mn\sqrt{4 \cdot 22 \cdot m^2 \cdot m \cdot n^6 \cdot n}$ $= -4mn\sqrt{4}\sqrt{22}\sqrt{m^2}\sqrt{m}\sqrt{n^6}\sqrt{n}$ $= -4mn \cdot 2 \cdot \sqrt{22} \cdot m \cdot \sqrt{m} \cdot n^3 \cdot \sqrt{n}$ $= -8m^2n^4\sqrt{22mn}$