Key

Sect 9.6
Exponential Growth and Decay Equation development

What is the exponential equation you learned at the beginning of this chapter? $y = a \cdot b^x$

How can we relate this equation to any growth or decay problem? $b > 1$, GROWTH / $b < 1$, DECAY

Initial value y_0	Multiplier	First three values after initial value y_1	y_2	y_3	% increase or decrease $\frac{\text{difference}}{\text{initial value}}$	Equation
20	1.5	30	45	67.5	$\frac{10}{20} = +.5$	$y = 20(1.5)^x$
100	1.25	125	156.25	195.3125	$\frac{25}{100} = +.25$	$y = 100(1.25)^x$
10	1.65	16.5	27.225	44.92125	$\frac{6.5}{10} = +.65$	$y = 10(1.65)^x$
16	1.04	16.64	17.3056	17.9978	+ 4%	$y = 16(1.04)^x$
500	0.2	100	20	4	$\frac{-400}{500} = -.8$	$y = 500(.2)^x$ OR $y = 500(1-.8)^x$
120	0.75	90	67.5	50.625	$\frac{-30}{120} = -.25$	$y = 120(.75)^x$ OR $y = 120(1-.25)^x$
50	.80	40	32	25.6	- 20%	$y = 50(.80)^x$ OR $y = 50(1-.20)^x$

Exponential Growth Formula:		Exponential Decay Formula:
$y = a(1+r)^t$	a = initial amount r = rate of change t = time y = ending amount	$y = a(1-r)^t$

Example 1: Tony has been offered a job with a starting salary of $40,000. Each year, he will receive a 4.5% raise.

$y = 40000(1+.045)^t$

a) Use the equation to figure out how much money Tony will earn after 10 years on the job.

$40000(1+.045)^{10} = \boxed{\$62,118.78}$

b) In how many years will Tony earn $100,000?

$100000 = 40000(1.045)^t$

$2.5 = (1.045)^t$

$\log 2.5 = t \cdot \log 1.045$

$t = \boxed{20.82 \text{ years}}$

Example 2: The city of Cleveland, OH had a population of 915,000 in 1950 and in 2010 only had a population 430,000 people.

not solving for the exponent, DON'T NEED TO USE LOGS!

a. If the population decreased exponentially, what is the rate at which it decreased per year?

$430000 = 915000(1-r)^{60}$

$(.4699)^{\frac{1}{60}} = ((1-r)^{60})^{\frac{1}{60}}$

$.9875 = 1-r$

$r = .0125$

$\boxed{1.25\% \text{ decrease}}$

b. Using the rate in part a, when will the population reach 350,000 people?

$350000 = 915000(1-.0125)^t$

$.3825 = (.9875)^t$

$\log .3825 = t \cdot \log .9875$

$\boxed{t = 76.4 \text{ years}}$

Example 3: You try☺ The population of a city of one million is increasing at a rate of 3% per year. It the population continues to grow at this rate, in how many years will the population have doubled?

$2000000 = 1000000(1+.03)^t$

$2 = (1.03)^t$

$\log 2 = t \cdot \log 1.03$

$\boxed{23.45 = t \text{ years}}$