9.5 Base *e* and Natural Logs Name Key Date________
Algebra 2 Trig

The other day, we compared different compoundings of a \$1000 investment with a 1.5% annual interest rate. After 2 years of growth we get the following results:

* There's a limit to the amount based on the number of compoundings.

- Compounded **annually**: $y = 1000(1+.015)^2 = \$1030.22$
- Compounded **quarterly**: $y = 1000\left(1+\frac{.015}{4}\right)^8 = \$1030.39$
- Compounded **monthly**: $y = 1000\left(1+\frac{.015}{12}\right)^{24} = \$1030.43$
- Compounded **daily**: $y = 1000\left(1+\frac{.015}{365}\right)^{730} = \$1030.45$
- Compounded **every second**: $y = 1000\left(1+\frac{.015}{31536000}\right)^{63072000} = \$1030.45$
- Compounded **continuously**: y = $1000e^{.015(2)} = \$1030.45$

$\boxed{A = Pe^{rt}}$ A = Ending acct value
P = Principal t = years
r = rate (in decimal)

Base *e* and Natural Logs

***e* is an irrational number like pi**

$e =$ 2.71828...

A logarithm with a base of *e* is called a natural logarithm.

$\boxed{\log_e x = \ln x}$

1) Rewrite in exponential form.

a) $\ln 5 \approx 1.609$

$\log_e 5 \approx 1.609$

$\boxed{e^{1.609} \approx 5}$

b) $\ln e = 1$

$\log_e e = 1$

$\boxed{e^1 = e}$

2) Rewrite in logarithmic form.

a) $e^3 \approx 20.09$

$\log_e 20.09 \approx 3$

$\boxed{\ln 20.09 \approx 3}$

b) $e^{-2} \approx 0.135$

$\log_e 0.135 \approx -2$

$\boxed{\ln 0.135 \approx -2}$

3) Solve for x.

a) $x = \ln e^4$

$x = \log_e e^4$

$e^x = e^4$

$\boxed{x=4}$

b) $x = \ln 1$

$x = \log_e 1$

$e^x = 1$

$\boxed{x=0}$

c) $5e^x + 12 = 62$

$5e^x = 50$

$e^x = 10$

$x \cdot \log e = \log 10$

$x = \frac{\log 10}{\log e} = \boxed{2.30}$

d) $2e^{3x} - 8 = 6$

$2e^{3x} = 14$

$e^{3x} = 7$

$3x \cdot \log e = \log 7$

$3x = \frac{\log 7}{\log e}$

$\boxed{x = .65}$

e) $24 - \frac{2}{3}e^{2x} = 18$

$-\frac{2}{3}e^{2x} = -6$

$e^{2x} = 9$

$2x \cdot \log e = \log 9$

$2x = \frac{\log 9}{\log e}$

$\boxed{x = 1.099}$

4) Use the formula $\boxed{A = Pe^{rt}}$ where *A* is the amount in the account after *t* years. *P* is the principal (the original amount) and *r* is the annual interest rate (in decimal form).

Suppose you deposit \$3000 in an account paying 2.8% annual interest compounded continuously.

a) What is the balance after 8 years?

$A = 3000e^{.028(8)}$

$A = \boxed{\$3753.21}$

b) How long will it take for the balance of the account to *double*?

$6000 = 3000e^{.028t}$

$2 = e^{.028t}$

$\log 2 = .028t \cdot \log e$

$.6931 = .028t$

$\boxed{24.76 = t \text{ years}}$