

Key

Section 9.3 – Properties of Logarithms Day 1

Remember how to simplify these?

1) $x^2 \cdot x^4 = x^{2+4} = x^6$

2) $\frac{x^5}{x^3} = x^{5-3} = x^2$

REMEMBER: Logarithms are exponents

Property I (on calculator)

$\log 7 = .845$

$\log 2 = .301$

$\log 14 = 1.146$

What do you notice about $\log 7$ and $\log 2$ in relation to $\log 14$? $\log 7 + \log 2 = \log 14$

So, $\log(7 \cdot 2) = \log 7 + \log 2$

When you multiply two powers with the same base, you add the logs.

Logarithm of a Product
 $\log(xy) = \log x + \log y$

Directions: Use the log of a product property to write the log as the sum of two logs...

example

Ex1) $\log 6 = \log 2 + \log 3$

Ex2) $\log 26 = \log 2 + \log 13$

Property II Use your calculator to evaluate:

a) $\log 22 = 1.342$

b) $\log 2 = .301$

c) $\log 11 = 1.041$

What did you notice about $\log 22$ and $\log 2$ in relation to $\log 11$? subtraction

So, $\log\left(\frac{22}{2}\right) = \log 22 - \log 2$

Because, LOGARITHMS ARE exponents! When you divide two powers with the same base, you subtract the logs.

Logarithm of a Quotient
 $\log\left(\frac{x}{y}\right) = \log x - \log y$

Use the log of a quotient property to write the log as the difference of two logs...

$$\text{Ex1) } \log\left(\frac{3}{2}\right) = \log 3 - \log 2$$

$$\text{Ex2) } \log 2.5 = \log\left(\frac{5}{2}\right) = \log 5 - \log 2$$

Property III. Use your calculator to evaluate:

$$\log 8 \quad \underline{\log(2 \cdot 2 \cdot 2)}$$

$$3(\log 2) \quad \underline{\log 2 + \log 2 + \log 2}$$

$$\log 8 = \log(2^3) = 3(\log 2)$$

Logarithm of a Power

$$\log_b(x^n) = n \cdot \log_b x$$

Let's try some of these... write these logarithms with the exponent of the product in front of the log.

$$1) \log 16 = \log 2^4 \\ = 4(\log 2)$$

$$2) \log 36 = \log 6^2 \\ = 2(\log 6)$$

$$3) \log 216 = \log 6^3 \\ = 3(\log 6)$$

One more things to remember...

Can you get the results above from other bases, besides base 10? yes!

Let's give it a try☺ Simplify...

$$\log_4 4 = \underline{1}$$

$$\log_5 5 = \underline{1}$$

$$\log_5 25 = \underline{2}$$

How can these help you? You will see in the next problems☺

Break it down!

Now let's try some. Evaluate each log by letting $\log_5 2 = .4307$, $\log_5 3 = .6826$.

Also remember that $\log_5 5 = 1$

$$\begin{aligned} 1) \log_5 6 &= \log_5 2 + \log_5 3 \\ &= .4307 + .6826 \\ &= \boxed{1.1133} \end{aligned}$$

$$\begin{aligned} 2) \log_5 20 &= \log_5 5 + \log_5 2^2 \\ &= \log_5 5 + 2(\log_5 2) \\ &= 1 + 2(.4307) \\ &= \boxed{1.8614} \end{aligned}$$

$$\begin{aligned} 3) \log_5 25 &= \log_5 5^2 \\ &= 2(\log_5 5) \\ &= 2(1) \\ &= \boxed{2} \end{aligned}$$

$$\begin{aligned} 4) \log_5 3^5 &= 5(\log_5 3) \\ &= 5(.6826) \\ &= \boxed{3.413} \end{aligned}$$

$$\begin{aligned} 5) \log_5 \left(\frac{3}{5}\right) &= \log_5 3 - \log_5 5 \\ &= .6826 - 1 \\ &= \boxed{-.3174} \end{aligned}$$

$$\begin{aligned} 6) \log_5 100 &= \log_5 5^2 + \log_5 2^2 \\ &= 2 + 2(.4307) \\ &= \boxed{2.8614} \end{aligned}$$

What if we did these same problems, but let $\log_5 2 = A$, $\log_5 3 = B$ and $\log_5 5 = 1$. How would your answers change??

$$\begin{aligned} 1) \log_5 6 &= \log_5 2 + \log_5 3 \\ &= \boxed{A + B} \end{aligned}$$

$$\begin{aligned} 2) \log_5 20 &= \log_5 5 + 2(\log_5 2) \\ &= \boxed{1 + 2A} \end{aligned}$$

$$\begin{aligned} 3) \log_5 25 &= \log_5 5^2 \\ &= \boxed{2} \end{aligned}$$

$$\begin{aligned} 4) \log_5 3^5 &= 5(\log_5 3) \\ &= \boxed{5B} \end{aligned}$$

$$\begin{aligned} 5) \log_5 \left(\frac{3}{5}\right) &= \log_5 3 - \log_5 5 \\ &= \boxed{A - 1} \end{aligned}$$

$$\begin{aligned} 6) \log_5 100 &= \log_5 5^2 + \log_5 2^2 \\ &= \boxed{2 + 2A} \end{aligned}$$

Find the logarithm without using your LOG key on your calculator! You must have no logs in your answer. Let $\log 2 = a$, $\log 3 = b$ and $\log 5 = c$. Look for base 10 logs!

$$\begin{aligned} 1) \log 6 &= \log 2 + \log 3 \\ &= \boxed{a + b} \end{aligned}$$

$$\begin{aligned} 2) \log 200 &= \log 10^2 + \log 2 \\ &= \boxed{2 + a} \end{aligned}$$

$$\begin{aligned} 3) \log \left(\frac{3}{2}\right) &= \log 3 - \log 2 \\ &= \boxed{b - a} \end{aligned}$$

$$\begin{aligned} 4) \log 2^7 &= 7(\log 2) \\ &= \boxed{7a} \end{aligned}$$

$$\begin{aligned} 5) \log 16 &= \log 2^4 \\ &= 4(\log 2) \\ &= \boxed{4a} \end{aligned}$$

$$\begin{aligned} 6) \log \frac{1}{9} &= \log 1 - \log 3^2 \\ &= 0 - 2(b) \\ &= \boxed{-2b} \end{aligned}$$