Key

Section 9.2 – Logarithms – Day 1

$$\mathbf{y = \log_b x \quad \text{if and only if} \quad b^y = x}$$

where $x > 0$, $b > 0$ and $b \neq 1$

There are two forms. Being able to switch from one form to the other is important!

Logarithmic form

argument

$\log_b x = y$

exponent

base

Exponential form

exponent

$b^y = x$

argument

base

A logarithm is an EXPONENT

When we evaluate $\log_3 9$ we are looking for "what exponent goes on" 3 to get 9?

$\log_3 9$

WEGO $_3 9 = \boxed{2}$

Think: **WEGO!!!!!**

The key to working with logarithms is to think of them as exponents!!!!

Type A In these problems below we are looking for an exponent
Use *WEGO* to help you☺

1. $\log_5 125$ $(5^x = 125)$

wego $_5 125$

$\boxed{3}$

2. $\log_4 \frac{1}{16}$ $(4^x = \frac{1}{16})$

wego $_4 \frac{1}{16}$

$\boxed{-2}$

3. $\log_2 32$ $(2^x = 32)$

wego $_2 32$

$\boxed{5}$

4. $\log_y (y^2)$

wego $_y y^2$

$\boxed{2}$

5. $\log_3 \frac{1}{3}$ $(3^x = \frac{1}{3})$

wego $_3 \frac{1}{3}$

$\boxed{-1}$

6. $\log_1 4$ $(1^x = 4)$ $b \neq 1$ *

wego $_1 4$

$\boxed{\text{no solution}}$

There are three basic types of problems.

1) **Type A: finding the logarithms (the exponenet) (we just did these:)**
2) **Type B: Finding the argument**
3) **Type C: Finding the base**

Type B: Finding the argument

Example: $\log_2 x = 4$ **Procedure:** put them in exponential form

Try:

1. $\log_2 x = 6$

$2^6 = x$

$\boxed{64 = x}$

2. $\log_{27} x = -\frac{1}{3}$

$27^{-1/3} = x$

$(3^3)^{-1/3} = x$

$3^{-1} = x$

$\boxed{\frac{1}{3} = x}$

3. $\log_5 x = -3$

$5^{-3} = x$

$\frac{1}{5^3} = x$

$\boxed{\frac{1}{125} = x}$

4. $\log_{81} x = \frac{1}{4}$

$81^{1/4} = x$

$(3^4)^{1/4} = x$

$\boxed{3 = x}$

Type C: Finding the base

Example: $\log_x 81 = 4$ **Procedure:** ① put in exponential form ② raise your variable to the reciprocal

1. $\log_x \frac{1}{25} = \frac{1}{2}$

$(x^{1/2})^2 = \left(\frac{1}{25}\right)^2$

$\boxed{x = \frac{1}{625}}$

2. $\log_x 16 = -4$

$(x^{-4})^{-1/4} = (16)^{-1/4}$

$x = (2^4)^{-1/4}$

$x = 2^{-1}$

$\boxed{x = \frac{1}{2}}$

5. $\log_x 64 = 3$

$(x^3)^{1/3} = (64)^{1/3}$

$x = (4^3)^{1/3}$

$\boxed{x = 4}$

4. $\log_x \frac{1}{125} = -3$

$(x^{-3})^{-1/3} = \left(\frac{1}{125}\right)^{-1/3}$

$x = \left(\left(\frac{1}{5}\right)^3\right)^{-1/3}$

$x = \left(\frac{1}{5}\right)^{-1}$

$\boxed{x = 5}$

What if you saw something like… log 100. What's missing? the base

Whenever there is no base written, assume it is 10

Therefore, log 100 is really $\log_{10} 100$ and the result is 2.

$(10^x = 100)$

Try some!

1. $\log_{10} 10$ $\quad 10^x = 10$

$\boxed{1}$

2. $\log \frac{1}{10}$ $\quad 10^x = \frac{1}{10}$

$\boxed{-1}$

3. log 10,000 $\quad 10^x = 10000$

$\boxed{4}$

4. $\log \frac{1}{1000}$ $\quad 10^x = \frac{1}{1000}$

$\boxed{-3}$