7.6 - Rational Exponents!

Key

$b^{\frac{1}{n}} = \sqrt[n]{b^1}$
$b^{\frac{m}{n}} = \sqrt[n]{b^m}$

* the index goes in the denominator

Write in radical form, and simplify if possible.

1. $28^{\frac{1}{2}} = \sqrt[2]{28} = \sqrt[2]{(2)^2 \cdot 7} = \boxed{2\sqrt{7}}$

2. $25^{\frac{3}{2}} = \sqrt[2]{25^3} = \sqrt[2]{25^2 \cdot 25} = \sqrt[2]{25^2 \cdot 5^2} = 25 \cdot 5 = \boxed{125}$

3. $\left(x^{\frac{2}{1}}\right)^{\frac{5}{3}} = x^{\frac{10}{3}} = \sqrt[3]{x^{10}} = \sqrt[3]{(x^3)^3 \cdot x} = \boxed{x^3\sqrt[3]{x}}$

Write each radical using rational exponents, and simplify if possible.

1. $\sqrt[2]{9} = 9^{1/2} = (3^2)^{1/2} = \boxed{3}$

2. $\sqrt[3]{b^2} = \boxed{b^{2/3}}$

3. $\sqrt[4]{16^3} = 16^{3/4} = (2^4)^{3/4} = 2^3 = \boxed{8}$

Evaluate each expression. Use this method:

$-b^{\frac{-m}{n}} = -\left(\left(b\right)^{\frac{m}{n}}\right)^{-1}$	Remember to break down the base!!

1. $64^{\frac{5}{6}} = (2^6)^{5/6} = 2^5 = \boxed{32}$

2. $-27^{\frac{2}{3}} = -(3^3)^{2/3} = -(3^2) = \boxed{-9}$

3. $27^{-\frac{2}{3}} = \left((27)^{\frac{2}{3}}\right)^{-1} = \left((3^3)^{\frac{2}{3}}\right)^{-1} = (3^2)^{-1} = 9^{-1} = \boxed{\frac{1}{9}}$

4. $\left(\frac{-8}{-125}\right)^{\frac{1}{3}} = \left(\frac{8}{125}\right)^{1/3} = \left(\left(\frac{2}{5}\right)^3\right)^{\frac{1}{3}} = \boxed{\frac{2}{5}}$

5. $\left(-\frac{125}{27}\right)^{\frac{2}{3}} = \left(\left(\frac{-5}{3}\right)^3\right)^{\frac{2}{3}} = \left(\frac{-5}{3}\right)^2 = \boxed{\frac{25}{9}}$

6. $\left(-\frac{125}{27}\right)^{\frac{-2}{3}} = \left(\left(\left(\frac{-5}{3}\right)^3\right)^{2/3}\right)^{-1} = \left(\left(\frac{-5}{3}\right)^2\right)^{-1} = \left(\frac{25}{9}\right)^{-1} = \boxed{\frac{9}{25}}$

Simplifying Expressions with Rational Exponents

A rational exponent is simplified if…

1. It is not a complex fraction (the numerator, denominator or both contain a fraction).
2. The base is as low as possible.
3. It has no fractional exponents in the denominator
4. It's fractional exponent in the numerator is in lowest terms

Of course you need to multiply (#1), divide (#5), or take a power of a power(#3), before you can determine if the rational exponent needs to be simplified☺

Simplify each expression:

1. $y^{\frac{2}{3}} \bullet y^{\frac{3}{8}} = \boxed{y^{25/24}}$

$\frac{16}{24} + \frac{9}{24}$

2. $x^{-\frac{3}{4}} \bullet x^{\frac{4}{3}} = \boxed{x^{7/12}}$

$\frac{-9}{12} + \frac{16}{12}$

3. $\left(y^{\frac{2}{3}}\right)^{\frac{3}{4}} = y^{6/12}$

$= \boxed{y^{1/2}}$

4. $\left(m^{-\frac{6}{5}}\right)^{\frac{2}{5}} = \boxed{m^{-12/25}}$

5. $\dfrac{p^1}{p^{\frac{1}{3}}} = \boxed{p^{2/3}}$

$\frac{3}{3} - \frac{1}{3}$

6. $\sqrt[4]{25} \bullet \sqrt[5]{25}$

$25^{\frac{1}{4}} \cdot 25^{\frac{1}{5}} = 25^{9/20}$

$\frac{5}{20} + \frac{4}{20}$

$= (5^2)^{9/20}$

$= 5^{18/20}$

$= \boxed{5^{9/10}}$

7. $\sqrt[3]{25} \bullet \sqrt{125}$

$\sqrt[3]{5^2} \cdot \sqrt[2]{5^3}$

$5^{2/3} \cdot 5^{3/2}$

$\frac{4}{6} + \frac{9}{6} = \frac{13}{6}$

$\boxed{5^{13/6}}$

8. $\dfrac{a^{-\frac{1}{5}}}{a^{-\frac{11}{15}} \bullet a^{\frac{2}{5}}} = \dfrac{a^{-\frac{1}{5}}}{a^{-\frac{1}{3}}}$

$\frac{-11}{15} + \frac{6}{15} = \frac{-5}{15} = \frac{-1}{3}$

$= \boxed{a^{2/15}}$

$\frac{-3}{15} - \frac{-5}{15} = \frac{2}{15}$