Key

# 7.5 – Operations with Radical Expressions

## Simplifying Radical Expressions

A radical expression is in simplified form when:

- ☺ Exponent under the radical is less than the index $\sqrt[3]{x^2}$ ☺ $\sqrt[3]{x^7}$ ☹
- ☺ No factor is a perfect square, perfect cube, .... $\sqrt{100x}$
- ☺ No radicals in the denominator

### Examples

1. $\sqrt{25a^4b^9}$

$\sqrt[2]{5^2(a^2)^2(b^4)^2\cdot b}$

$\boxed{5a^2b^4\sqrt{b}}$

2. $\sqrt{16p^8q^7}$

$\sqrt[2]{4^2(p^4)^2(q^3)^2\cdot q}$

$\boxed{4p^4q^3\sqrt{q}}$

3. $\sqrt[3]{8x^3y^7z^{12}}$

$\sqrt[3]{(2)^3(x)^3(y^2)^3\cdot y\cdot(z^4)^3}$

$\boxed{2xy^2z^4\sqrt[3]{y}}$

4. $\sqrt[3]{27a^9b^3c^4}$

$\sqrt[3]{3^3(a^3)^3(b)^3(c)^3\cdot c}$

$\boxed{3a^3bc\sqrt[3]{c}}$

5. $\sqrt[3]{16x^2y^6z^{10}}$

$\sqrt[3]{(2)^3\cdot 2(x)^2(y^2)^3(z^3)^3 z}$

$\boxed{2y^2z^3\sqrt[3]{2x^2z}}$

6. $\sqrt[4]{16x^5y^4z^9}$

$\sqrt[4]{2^4(x)^4\cdot x\cdot(y)^4\cdot(z^2)^4\cdot z}$

$\boxed{2xyz^2\sqrt[4]{xz}}$

## Rationalizing the Denominator

1. $\sqrt{\dfrac{x^4}{y^5}} = \dfrac{\sqrt[2]{x^4}}{\sqrt[2]{y^5}} = \dfrac{x^2}{y^2\sqrt{y}}\cdot\dfrac{\sqrt{y}}{\sqrt{y}} = \boxed{\dfrac{x^2\sqrt{y}}{y^3}}$

$\sqrt[2]{(y^2)^2\cdot y}$ $\quad \underbrace{\sqrt{y}\cdot\sqrt{y}}_{y}$

2. $\sqrt{\dfrac{40}{10}} = \sqrt{4} = \boxed{2}$

3. $\sqrt[2]{\dfrac{y^8}{x^7}} = \dfrac{\sqrt[2]{y^8}}{\sqrt[2]{x^7}} = \dfrac{y^4}{x^3\sqrt{x}}\cdot\dfrac{\sqrt{x}}{\sqrt{x}} = \boxed{\dfrac{y^4\sqrt{x}}{x^4}}$

$\sqrt[2]{(x^3)^2\cdot x}$ $\quad \underbrace{\sqrt{x}\cdot\sqrt{x}}_{x}$

Operations with Radicals

## Add and Subtract Radicals

"Like radicals" - index and radicand must be alike.

Like Radicals

$5\sqrt[3]{2a}$ and $-10\sqrt[3]{2a}$

NOT Like Radicals

$5\sqrt[4]{2a}$ and $5\sqrt[3]{2a}$

1. $3\sqrt{45}-5\sqrt{80}+4\sqrt{20}$

$3\cdot\sqrt{9}\cdot\sqrt{5}-5\sqrt{16}\cdot\sqrt{5}+4\cdot\sqrt{4}\sqrt{5}$

$9\sqrt{5}-20\sqrt{5}+8\sqrt{5}$

$\boxed{-3\sqrt{5}}$

2. $2\sqrt[3]{16}-5\sqrt[3]{2}+\sqrt[3]{54}-\sqrt[3]{24}$

$2\sqrt[3]{2^3\cdot2}-5\sqrt[3]{2}+\sqrt[3]{3^3\cdot2}-\sqrt[3]{2^3\cdot3}$

$4\sqrt[3]{2}-5\sqrt[3]{2}+3\sqrt[3]{2}-2\sqrt[3]{3}$

$\boxed{2\sqrt[3]{2}-2\sqrt[3]{3}}$

## Multiply Radicals

1. $6\sqrt[3]{9n^2}\bullet3\sqrt[3]{24n}=18\sqrt[3]{216n^3}$

$=18\sqrt[3]{6^3n^3}$

$=18\cdot6\cdot n$

$=\boxed{108n}$

2. $5\sqrt[4]{24x^3}\bullet4\sqrt[4]{54x}=20\sqrt[4]{1296x^4}$

$=20\sqrt[4]{6^4\cdot x^4}$

$=20\cdot6\cdot x$

$=\boxed{120x}$

3. $(3\sqrt{5}-2\sqrt{3})(2+\sqrt{3})$

$6\sqrt{5}+3\sqrt{15}-4\sqrt{3}-2\sqrt{9}$

$\boxed{6\sqrt{5}+3\sqrt{15}-4\sqrt{3}-6}$

4. $(5\sqrt{3}-6)(5\sqrt{3}+6)$

$25\sqrt{9}+\cancel{30\sqrt{3}}-\cancel{30\sqrt{3}}-36$

$75-36$

$\boxed{39}$

## Dividing Radicals – using a Conjugate to Rationalize a Denominator

1. $\frac{1-\sqrt{3}}{5+\sqrt{3}}\cdot\frac{(5-\sqrt{3})}{(5-\sqrt{3})}=\frac{5-\sqrt{3}-5\sqrt{3}+3}{25-5\sqrt{3}+5\sqrt{3}-3}$

$=\frac{8-6\sqrt{3}}{22}$

$=\boxed{\frac{4-3\sqrt{3}}{11}}$

2. $\frac{4+\sqrt{2}}{5-\sqrt{2}}\cdot\frac{(5+\sqrt{2})}{(5+\sqrt{2})}=\frac{20+4\sqrt{2}+5\sqrt{2}+2}{25+5\sqrt{2}-5\sqrt{2}-2}$

$=\boxed{\frac{22+9\sqrt{2}}{23}}$

3. $\frac{3-2\sqrt{5}}{6+\sqrt{5}}\cdot\frac{(6-\sqrt{5})}{(6-\sqrt{5})}=\frac{18-3\sqrt{5}-12\sqrt{5}+10}{36-6\sqrt{5}+6\sqrt{5}-5}$

$=\boxed{\frac{28-15\sqrt{5}}{31}}$

4. $\frac{1+2\sqrt{3}}{3-3\sqrt{2}}\cdot\frac{(3+3\sqrt{2})}{(3+3\sqrt{2})}=\frac{3+3\sqrt{2}+6\sqrt{3}+6\sqrt{6}}{9+9\sqrt{2}-9\sqrt{2}-18}$

$\frac{3+3\sqrt{2}+6\sqrt{3}+6\sqrt{6}}{-9}=\boxed{\frac{1+\sqrt{2}+2\sqrt{3}+2\sqrt{6}}{-3}}$